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DECREASE OF RESISTANCE IN A MICROPOLAR LIQUID

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Modified equations of motion of a micropolar liquid are derived. It is shown that, in nonlinear problems, a lower resistance is possible in comparison with an ordinary liquid, even in the case of laminar flow.

There are many ways to describe rheologically complex liquids, where the asymmetry of the stress tensor and the deformation rate tensor (viscoelastic liquids), the relaxation terms (Maxwellian liquid), etc. are accounted for to a greater or lesser degree. The phenomenological derivation of the equations [1] where only the asymmetry of the stress tensor is considered already leads to an extremely complex system of 19 partial differential equations with 22 viscosity coefficients. Introduction of the additional assumption of isotropy made it possible to reduce this system to seven equations [2, 3], which are at present widely used for describing liquids with polymer additions, liquid crystals, blood, etc. (see, for instance, surveys [4, 5]).

Asymmetric hydromechanics [2, 3] is characterized by the nonsymmetric stress tensor σ_{ij} and the additional tensor of micromoments m_{ij}:

$$\sigma_{ij} = -p\delta_{ij} + \mu(\partial_i V_j + \partial_j V_i) + k(\partial_i V_j - \varepsilon_{ijm}\Omega_m), \tag{1}$$

$$m_{ij} = \alpha\delta_{ij}\operatorname{div}\Omega + \beta\partial_j\Omega_i + \gamma\partial_i\Omega_j,\ \partial_i = \partial/\partial x_i. \tag{2}$$

Thus, besides the coefficient of dynamic viscosity μ, there are in asymmetric hydrodynamics three additional rotational viscosity coefficients and the coefficient k, which provides the measure of a particle's "coupling" with its ambient. The dilatational viscosity coefficient does not figure in (1), since we limit our considerations here to the case of incompressible liquids. It is evident from (1) and (2) that, in Eulerian presentation, a state in asymmetric hydromechanics is determined not only be the field of velocities V, but also by the field of angular velocities of microrotation Ω.

The equations of motion in asymmetric hydromechanics are given by

$$\rho\frac{dV}{dt} = (\mu + k)\Delta V + k\operatorname{rot}\Omega - \nabla p,\ \operatorname{div} V = 0, \tag{3}$$

$$\rho J\frac{d\Omega}{dt} = (\alpha + \beta)\operatorname{grad}(\operatorname{div}\Omega) + \gamma\Delta\Omega - 2k\Omega + k\operatorname{rot} V. \tag{4}$$

The microinertial characteristics of the medium were not taken into account in [2] [the left-hand side of Eq. (4) was assumed to be zero], while Eq. (4) was used in [3] and the subsequently published papers (see the literature cited in [4, 5]).

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We shall show that Eq. (4) does not satisfy the basic requirements imposed on the evolution of axial vector fields in three-dimensional space. For this, we shall consider the much more general problem concerning the evolution of the differential q-form Φ under the action of viscosity and the medium's motion, assigned by the vector field V. The corresponding equation is given by [6]

$$\left(\frac{\partial}{\partial t} + L_V\right)\Phi = \text{viscous terms,} \tag{5}$$

where L_V is the Lie differential operator along the field V. For q = 0 (i.e., for the functions) and q = 1 (i.e., for the vector fields), the operator on the left-hand side of (5) coincides with the total derivative operator $d/dt = \partial/\partial t + (V\nabla)$. For q = 2, the operator type depends on the dimensionality of the space; in the three-dimensional case 2 in which we are interested, the form $\Phi = P dy\wedge dz + Q dz\wedge dx + R dx\wedge dy$ can be juxtaposed with the vector field Ω, whose components are P, Q, and R (the axial vector in the physical terminology), so that (5) assumes the following form:

$$\frac{\partial \Omega}{\partial t} - \operatorname{rot}[V\times\Omega] + V\operatorname{div}\Omega = \text{viscous terms.} \tag{6}$$

It is clear from the above that the evolution of V is actually described by Eq. (3) [the viscous terms are calculated by means of (1)], while the evolution of Ω must be described by the modified equation

$$\rho J\left[\frac{d\Omega}{dt} - (\Omega\nabla)V\right] = (\alpha+\beta)\operatorname{grad}(\operatorname{div}\Omega) + \gamma\Delta\Omega + k(\operatorname{rot}V - 2\Omega). \tag{7}$$

The left-hand side of (7) follows immediately from (6) with an allowance for the incompressibility of the medium ($\operatorname{div}V = 0$), while the viscous terms in (7) are obtained directly from (2). It should be emphasized that, in contrast to (4), expression (7) coincides formally with an accuracy to the viscous terms with Helmholtz' equation for the evolution of the vorticity vector $\omega = \operatorname{rot}V$ in classical hydrodynamics [7]. This is natural, since both ω and Ω are axial vectors.

Using the terminology adopted in most papers, we shall refer to system (3)-(4) as the equations of a micropolar liquid (MPE); correspondingly, we shall refer to system (3), (7) as the modified equations of a micropolar liquid (MMPE). Let us now compare the MPE and MMPE characteristics by using examples of analytical solutions of these equations. The exact solutions of MPE are known for the two-dimensional and axisymmetric cases, and also for the limiting case of small Reynolds numbers [2-5]. It is readily seen that, in all these cases, MPE and MMPE are identically equal, and the difference between these equations manifests itself only in the essentially three-dimensional problem. The analogy with the classical case of a viscous, incompressible liquid suggests that, essentially, the unique example of this type where one could hope for an exact solution is the Kármán problem of rotation of an infinite disk [7]. We shall use the disk plane as the z = 0 plane of the cylindrical coordinate system (r, φ, z). We seek the solution of both the MPE and MMPE systems in the following form:

$$V = [r\omega F(\zeta),\ r\omega G(\zeta),\ \omega\lambda^{-1}H(\zeta)], \tag{8}$$

$$\Omega = [r\omega\lambda f(\zeta),\ r\omega\lambda g(\zeta),\ \omega h(\zeta)]. \tag{9}$$

Here, ω is the angular velocity of the disk, $\lambda = \sqrt{\omega\rho/(\mu + k)}$, and $\zeta = \lambda z$. Substitution of (8) and (9) in MPE and MMPE yields three coincident equations:

$$F^2 - G^2 + \frac{dF}{d\zeta}H - \frac{d^2F}{d\zeta^2} = C_1\frac{dg}{d\zeta},\quad 2FG + H\frac{dG}{d\zeta} - \frac{d^2G}{d\zeta^2} = C_1\frac{df}{d\zeta},$$
$$2F + \frac{dH}{d\zeta} = 0,\quad C_1 = k/(\mu + k) \tag{10}$$

and three additional equations for MPE,

$$C_4\left(Ff - Gg + H\frac{df}{d\zeta}\right) = C_2\frac{d^2f}{d\zeta^2} - C_1\left(2f + \frac{dG}{d\zeta}\right),$$

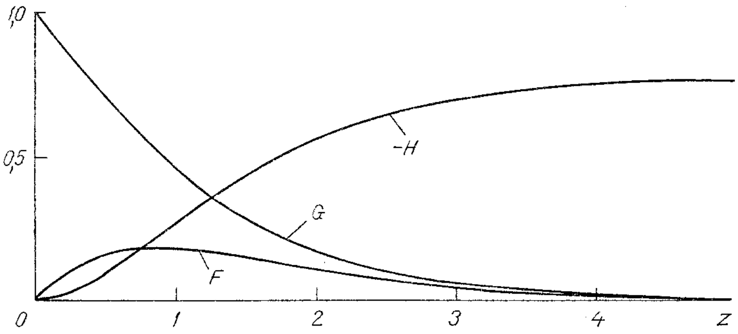

Fig. 1. Flow velocity components.

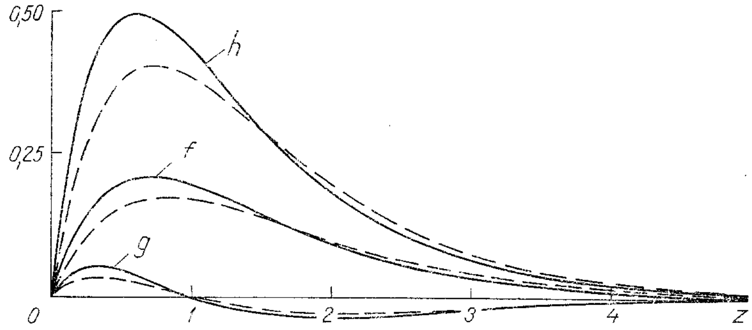

Fig. 2. Components of the angular velocity of microrotation. The solid curves represent calculations based on MPE, while the dashed curves pertain to calculations based on MMPE.

$$C_4\left(Fg+Gf+H\frac{dg}{d\zeta}\right)=C_2\frac{d^2g}{d\zeta^2}-C_1\left(2g-\frac{dF}{d\zeta}\right),$$

$$C_4\left(H\frac{dh}{d\zeta}\right)=C_3\left(\frac{d^2h}{d\zeta^2}+2\frac{df}{d\zeta}\right)-2C_2\frac{df}{d\zeta}-2C_1(h-G)\,, \tag{11}$$

and MMPE, respectively,

$$C_4\frac{d}{d\zeta}(fH-hF)=C_2\frac{d^2f}{d\zeta^2}-C_1\left(2f+\frac{dG}{d\zeta}\right),$$

$$C_4\left[\frac{d}{d\zeta}(gH-hG)-2(fG-gF)\right]=C_2\frac{d^2g}{d\zeta^2}-C_1\left(2g-\frac{dF}{d\zeta}\right), \tag{12}$$

$$C_4[2(hF-fH)]=C_3\left(\frac{d^2h}{d\zeta^2}+2\frac{df}{d\zeta}\right)-2C_2\frac{df}{d\zeta}-2C_1(h-G).$$

Here, $C_2 = \gamma\omega\rho(\mu + k)^{-2}$, $C_3 = (\alpha + \beta + \gamma)\omega\rho(\mu + k)^{-2}$, $C_4 = J\omega\rho(\mu + k)^{-1}$ are dimensionless constants.

The usual adhesion condition is used as the boundary condition for V. The problem of the boundary condition for Ω is less clear [8]. As an alternative to the adhesion condition $\Omega = 0$ [3], other possibilities have been discussed in the literature since the publication of [2], for instance, the condition $\Omega = 1/2\,\mathrm{rot}\,V$. For the sake of determinacy, we shall subsequently use the adhesion condition, although a solution can readily be obtained for other boundary conditions as well. By imposing also the usual damping conditions at infinity, we obtain the boundary conditions

$$F=H=0,\ G=1,\ f=g=h=0 \text{ for } \zeta=0, \tag{13}$$

$$F=G=0,\ f=g=h=0 \text{ for } \zeta=\infty. \tag{14}$$

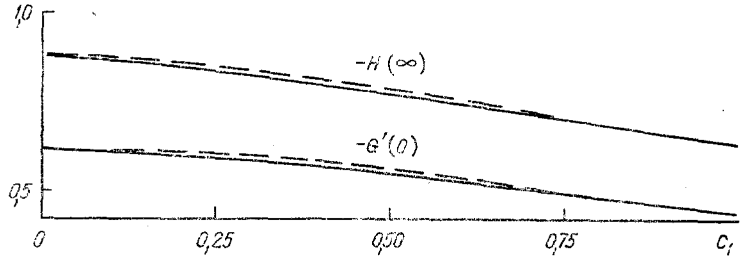

Fig. 3. Torque and axial velocity at infinity as functions of the coupling coefficient. The solid curves represent calculations based on MPE, while the dashed curves pertain to calculations based on MMPE.

Boundary-value problems for MPE (10), (11), (13), and (14) and MMPE (10) and (12)-(14) have been solved numerically by using the method of matrix trial runs. The calculation accuracy was checked by comparison with the classical Kármán solution [7], which corresponded to $C_1 = 0$. Figures 1 and 2 show the diagrams of the functions F, G, H, f, g, and h for $C_1 = 0.5$, $C_2 = C_3 = 0.25$, and $C_4 = 5 \cdot 10^{-3}$. The results of calculations based on MPE are indicated by solid curves, while the results obtained by means of MMPE are shown by dashed curves. In this case, the velocity is not greatly affected by changes in the equations (Figs. 1 and 3); the angular velocity of microrotation is affected more substantially (Fig. 2).

The resistance moment acting on the disk is determined by the means of the expression

$$M = 2\int_0^\infty (r\sigma_{\varphi z} + m_{\varphi z})\, 2\pi r dr. \tag{15}$$

Using the expressions for the stress tensor (1), the micromoment tensor (2) and the adhesion boundary conditions, we find

$$\sigma_{\varphi z} = \mu \frac{\partial V_\varphi}{\partial z}\bigg|_{z=0} = \mu\omega \sqrt{\frac{\omega}{\mu + k}}\, r \frac{dG}{d\zeta}(0), \tag{16}$$

$$m_{\varphi z} = \beta \frac{\partial \Omega_\varphi}{\partial z}\bigg|_{z=0} = \beta\omega \frac{\omega}{\mu + k}\, r \frac{dg}{d\zeta}(0). \tag{17}$$

The total moment acting on an infinite disk is infinite. Therefore, we find the moment acting on the finite part of a disk whose radius is R; if we neglect the edge effects, we find that the same moment also acts on a finite disk with the radius R. Substituting (16) and (17) in (15), we obtain

$$M = 4\pi \left[\mu \sqrt{\frac{\omega^3}{\mu + k}} \frac{dG}{d\zeta}(0) \frac{R^4}{4} + \beta \frac{\omega^2}{\mu + k} \frac{dg}{d\zeta}(0) \frac{R^3}{3}\right]. \tag{18}$$

For large values of R, the basic contribution is provided by the first term in (18),

$$M \approx \pi R^4 \sqrt{\mu\omega^3}\sqrt{1 - C_1}\, \frac{dG}{d\zeta}(0).$$

Figure 3 shows the calculation results for the torque and the axial velocity at infinity in relation to the degree of "coupling" between particles and the ambient. In this, the dependences of the parameters C_2, C_3, and C_4 on C_1 stood out clearly: $C_2 = A(1 - C_1)^2$, $C_3 = B(1 - C_1)^2$, and $C_4 = C(1 - C_1)$. It was assumed that A = B = 1 and $C = 10^{-2}$ in these calculations. It is evident from Fig. 3 that the torque diminishes with an increase in C_1. Qualitatively, this result did not change with variations in A, B, and C in the ranges A, B $\in$ [1, 10] and $C \in [10^{-2}, 10^{-1}]$ in our calculations. At first glance, such torque behavior seems unexpected, since the presence of additional viscous dissipation mechanisms in a micropolar liquid should augment the friction forces, the resistance moments, etc. However, such "additive" considerations are valid only in situations where MPE or MMPE become linear. Actually, in the case of Poiseuille or Couette flow and other linear problems, the forces and the moments increase [2, 3]. The Kármán problem remains nonlinear for any, even very small, values of the angular velocity ω. This is due to the absence of a characteristic dimension and, consequently, the impossibility of introducing the Reynolds number. On the whole, the above results indicate that, even in the case of laminar flow of a micropolar liquid, in essentially nonlinear problems, diminution of forces and moments in comparison with the case of an ordinary viscous liquid is possible.

We have not come across a solution of the Kármán problem for a micropolar liquid in publications. We shall attempt to find the solutions of the other nonlinear problems in classical hydromechanics that would admit of self-similar solutions in an ordinary viscous liquid. In the two-dimensional case, MPE and MMPE coincide and are given by

$$\begin{gathered}\rho\,\frac{\partial(\psi,\ \Delta\psi)}{\partial(x,\ y)}+(\mu+k)\,\Delta^2\psi+k\Delta\sigma=0,\\ \rho J\,\frac{\partial(\sigma,\ \psi)}{\partial(x,\ y)}+k\Delta\psi+2k\sigma-\gamma\Delta\sigma=0.\end{gathered}\tag{19}$$

Here, $V = (\partial\psi/\partial y, -\partial\psi/\partial x, 0)$ and $\Omega = (0, 0, \sigma)$. In the problem of flow in the vicinity of a planar critical point, we naturally seek the solution in the following form:

$$\psi=\frac{ax}{\lambda}F(\lambda y),\ \sigma=a\lambda xG(\lambda y),\ \lambda=\sqrt{\frac{a\rho}{\mu}}.\tag{20}$$

The constant a determines the behavior of the solution at a location remote from the solid surface $\psi \to a\,xy$. In this, (19) is reduced to a system of two ordinary differential equations, since the analog of the Hiemenz solution also exists for a micropolar liquid. In the problem of a wedge source, the first of Eqs. (19), written in terms of polar coordinates (r, θ), suggests that the solution be sought in the following form:

$$\psi=\psi(\theta),\ \sigma=\sigma(\theta)/r^2.\tag{21}$$

Substitution of (21) in (19) results in an overdetermined system of three ordinary differential equations, since the second of Eqs. (19) yields two equations, as it contains terms of the type $r^{-2}f_1(\theta)$ and $r^{-4}f_2(\theta)$. One of these equations can be conveniently rewritten by using the old notation,

$$\Omega=\frac{1}{2}\operatorname{rot}V,\tag{22}$$

while the other two can be reduced to an overdetermined system of equations with respect to $U = \partial\psi/\partial\theta$:

$$\begin{gathered}\frac{d^2U}{d\theta^2}+4U+U^2\left(\mu+\frac{k}{2}\right)^{-1}=\text{const},\\ \frac{d^2U}{d\theta^2}+4U+J\gamma^{-1}U^2=\text{const}.\end{gathered}\tag{23}$$

It is evident from (22) and (23) that the analog of the Geoffrey–Hamel solution for a micropolar liquid has a self-similar character only if condition (22) at solid boundaries is satisfied instead of the adhesion condition $\Omega = 0$ and if there is a relationship between the density of the micromoment of inertia and the dissipative coefficients,

$$J=\gamma\left(\mu+\frac{k}{2}\right)^{-1}.\tag{24}$$

A similar investigation of exact axisymmetric solutions for a micropolar liquid yielded the following results: There exists an exact solution in the problem of the spatial critical point for any boundary conditions; in the problem of jet outflow in a submerged space, a solution is possible only under condition (24). It should be noted that the solutions of analogs of the Geoffrey–Hamel and Landau–Squire problems, if they exist, are obtained directly from the classical ones by substitution of $\mu + k/2$ for μ. Generally, nonexistence of a self-similar solution of these problems is connected with the fact that an "internal" dimensional length $\sqrt{\gamma/\mu}$ exists in a micropolar liquid, in contrast to a classical one, so that considerations of dimensionality do not imply that the solution must have a self-similar form of the type (21). In the Kármán and Hiemenz problems, there is originally a dimensional length ($\sqrt{\mu/\rho\omega}$ and $\sqrt{\mu/\rho a}$, respectively), and the appearance of an additional "internal" length does not alter the form of solutions (8), (9), and (20). In conclusion, it should be noted that, in the general three-dimensional case, if there is relationship (24) and condition (22) is satisfied at solid boundaries, we naturally assume, by analogy with exact solutions, that (22) holds throughout the flow field. In this case, MMPE are self-consistent, in contrast to MPE. Actually, (3) is then transformed into an ordinary Navier–Stokes equation with the viscosity coefficient $\mu + k/2$, while (7) is transformed into the Helmholtz equation for vorticity [in contrast to (7), expression (4) does not convert to the Helmholtz equation]. This indicates once again that the correct approach in describing the flow of a micropolar liquid must be based on MMPE.

NOTATION

σ_{ij}, stress tensor; m_{ij}, micromoment tensor; δ_{ij}, Kronecker tensor; ε_{ijk} Levi–Civita tensor; α, β, and γ, rotational viscosity coefficients; μ and k, viscosity coefficients; p, pressure, ρ, density; j, micromoment of inertia; V, velocity, Ω, angular velocity of microrotation; Φ, differential form; L_V, Lie operator; r, φ, z, cylindrical coordinates; ω, angular velocity of the disk; F, G, H, f, g, and h, dimensionless functions; C_1, C_2, C_3, and C_4, dimensionless constants; M, moment; ψ, stream function; a and λ, constants.

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CHARACTERISTICS OF THE RHEOLOGICAL BEHAVIOR OF ELECTROSENSITIVE DISPERSIONS OF DIFFERENT STRUCTURAL-RHEOLOGICAL TYPES

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A classification of electrosensitive dispersions of different structural-rheological types is made.

According to existing ideas electrorheological fluid systems (ERS), which are sensitive to the action of an electric potential, consist of dispersed compositions with a complicated formula, in which the solid phase (most often silicon dioxide) is insoluble in the dispersion medium – nonpolar organic substances, for example, oils. Such compositions also contain a number of other necessary components, in particular, stabilizers and activators. The stabilizers become adsorbed on the developed surface of the particles of the solid phase and encapsulate them, thereby preventing conglomeration and precipitation. The activators, by polarizing the particles, make ERS electrically active. Under the action of an external potential the particles of the solid phase, becoming dipoles, interact actively with one another. In the process they rotate, which rotation is recorded on speckle pictures, and move into the volume of the dispersion medium, and this motion is accompanied by entrainment of the medium and local turbulence. As the intensity of the electric action becomes stronger the particles of the solid phase are combined into separate associates and continuous fi-

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